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INTRODUCTION TO A NEW INTERPRETATION OF RELATIVITY IN A COMPLEX SPACE

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Abstract

In Einstein's Relativity, the 4th-dimensional space-time manifold considers future-pointing and past-pointing matter. An interpretation of relativity, both in the special-relativistic domain and in the general-relativistic domain, inside a complex $\mathbb{C}^4$ space, is here proposed in order to face the matter that, in Einstein's approach, time is understood as a dimension through which a given physical object can move in both directions: into past and into future, leading to the existence of positive and negative space-time intervals, which seems logically inconsistent. In particular, an approach to general relativity inside a pseudo-complex space is suggested, which provides an extension of the standard general relativity and a justification of its results in terms of fundamental energy density fluctuations of a three-dimensional quantum vacuum and can be discriminated from general relativity in situations which are near the size of a black hole.

Keywords: special relativity, general relativity, complex space, three-dimensional quantum vacuum, energy density fluctuations.

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1. Introduction

In special relativity, the four-vector is introduced in order to unify space-time coordinates x, y, z, and t into a single entity. The length of this four-vector, called the space-time interval, is shown to be invariant, which means the same for all inertial observers.

In general relativity, the four-vector measuring the "length" of a particle worldline is parametrized by its proper time and can be positive or negative and depends on the direction of motion in future or in past:

$$d\tau = \pm\sqrt{dx^{\mu}dx_{\mu}} \qquad (1),$$

where τ is proper time [1]. Equation (1) implies that solutions with a positive or a negative sign are equally valid. This implies that, in general relativity, velocities can be backwards in time or future-pointing four-vectors.

In light of the equivalence principle, the laws of physics are the same for future-pointing (τ^{+}) and past-pointing (τ^{-}) observers, and this means that the terms "past" and "future" turn out to do not have an absolute meaning, they only have a meaning relative to the reference frame of the observer. By following [1], one can thus show that past-pointing four-vectors cannot be dealt with consistently in general relativity unless one accepts that the stress-energy tensor created/observed by matter going backwards in time inevitably has a sign opposite to the stress-energy tensor created/observed by matter going forward in time. In the light of these conjectures, one can conclude that time reversed matter moves on a different branch with respect to future pointing matter. Both branches, which interact through repulsive gravity, are connected by sharing the same stress-energy tensor up to a change in sign. On a large scale, equal amounts of future-pointing and past-pointing matter should be expected. This means that our experience of time as always going forwards is due to the separation of past-pointing matter and future-pointing matter by gravity (a spontaneous local symmetry breaking): the mutual repulsion between future-pointing matter and past-pointing matter could result in voids which generate a dark gravitational halo around galaxies.

By considering the energy density E measured by an observer at some point of a material object

$$E = T^{\mu\nu}u_{\mu}u_{\nu} \quad (2)$$

one finds immediately that stress-energy tensor $T^{\mu\nu}$ changes sign when the observer has its four-velocity u_μ reversed, thus leading to a different geometry of space-time for a past-pointing observer with respect to a future pointing observer. As a consequence, one has contributions to the stress-energy tensor characterized by a opposite sign for past-pointing matter and future-pointing matter, thus leading to two different metrics, and geodesics, for general relativity. In the light of this consideration, a paradox therefore emerges that when either the source or the observer is represented by a past-pointing four vector, the stress-energy tensor has the opposite sign than the conventional one, and consequently a second, different metric, which affects the geodesic curves in a non-trivial way, is required in order to assure consistency.

In this paper, in order to avoid the impasse generated by the consideration of negative solutions of the equation (1) of proper time, which leads to a different expression – of opposite sign – for the stress-energy tensor and thus to the consideration of a different metric tensor (which means a violation of the principle of uniqueness of free fall, in the sense that, if one reverses the proper time, one obtains a reversal of the four-velocity, and thus a change in the state of motion), we suggest an alternative approach where the fundamental arena of processes is a complex space which is ultimately linked with a microscopic quantum vacuum condensate. The paper is structured as follows. In section 2, we will introduce our interpretation of special relativity inside a complex $\mathbb{C}^4$ space. In section 3 we will develop a version of general relativity in a complex space which can provide an extension of general relativity able to take account of strong gravitational fields. In section 4 we will analyse some predictions of our model of general relativity in a complex space. Finally, in section 5 we will summarize the main results of our model, by putting in evidence some issues that will have to be addressed.

2. Replacement of the Minkowski manifold $\mathbb{R}^4$ with the complex manifold $\mathbb{C}^4$ in special relativity

In the space-time model $\mathbb{R}^4$ the fourth coordinate X_4 is imaginary, X_1, X_2, X_3 are real coordinates. In the complex $\mathbb{C}^4$ space, the 4th coordinate Z_4 is a complex coordinate as the other three coordinates Z_1, Z_2, Z_3, every point of it has complex coordinates:

$$z_i = x_i + i\, y_i \qquad (3),$$

where (x_i, y_i) $(i = 1,2,3,4)$. In $\mathbb{C}^4$ space-time is the duration of a motion of a given physical object. $\mathbb{C}^4$ is time-invariant in the sense that time is not its dimension [2].

As the authors have shown in [3, 4], the Einstein space-time arena $\mathbb{R}^4$ can be replaced by a three-dimensional (3D) Euclidean space where time has a different ontological status with respect to the coordinates of space in the sense that must be considered merely as a mathematical quantity measuring the numerical order of material changes. In this background, one considers Galilean transformations between the spatial coordinates X, Y, Z and X', Y', Z' of two inertial observers

$$\begin{aligned} X' &= X - v \cdot \tau \\ Y' &= Y \\ Z' &= Z \end{aligned} \qquad (4)$$

(where v is the velocity of the moving observer O' measured by the stationary observer O and τ is the proper time of the observer O), and Selleri's formalism [5, 6] as regards the transformation of the rate of clocks:

$$\tau' = \sqrt{1 - \frac{v^2}{c^2}} \cdot \tau \qquad (5)$$

which shows clearly that the speed of the moving clock is not related to the spatial coordinates but depends only of the speed v of the moving observer. In light of equations (4) and (5), in our theory the standard Einstein postulates of special relativity can be therefore replaced by the following ones [3, 4]:

1. The velocity of light has the same value in all directions and in all inertial systems (postulate of the invariance of the velocity of light in the vacuum).
2. The fundamental arena of physical processes is a 3D Euclid space where time exists merely as a mathematical quantity measuring the numerical order of material changes (and which can be defined as the proper time of the observer under consideration). Given two inertial frames, the transformations between the spatial coordinates of these two systems are given by equation (4), while the transformation of the rate of clocks, namely between the proper times of the observes of these two inertial systems, is given by equation (5).

Now, in the fundamental complex manifold $\mathbb{C}^4$ we define the concept of physical duration of motion as a proper, "true", physical scale for the proper time τ, namely for the numerical order, on the basis of equation:

$$t = \tau\left(1 - \frac{v^2}{c^2}\right) + \frac{vX}{c^2} \quad (6).$$

The physical time *t*, defined by equation (6) represents the duration of material motion as measured by the stationary observer O in the rest inertial frame. Equation (6) indicates that the numerical order may be considered as the internal structure of the physical, measurable time intended as duration.

By starting from the hypothesis that physical time is a scaling function of the fundamental numerical order of material change expressed by equation (6) and that the numerical order corresponding to the velocity of the moving observer with respect to the rest frame determines a re-scaling of the position of the object into consideration in the moving frame, one can show that the standard Lorentz transformations of Einstein's special relativity concerning the temporal coordinate and the first coordinate derive directly from a more fundamental arena equipped with the laws (4) and (5).

In summary, we can say that, in our approach, the duration of material changes satisfying the standard Lorentz transformation for the temporal coordinate is a proper, physical scaling function which emerges from the more fundamental numerical order and determines itself a re-scaling of the position measured by the moving observer expressed by the standard Lorentz transformation for the first spatial coordinate. Special Relativity equipped with formalisms (4) and (5) can also describe successfully all phenomena in 3D Euclidean space previously described by classical special relativity, such as aberration of light, Doppler effect, Jupiter's satellites occultation and radar ranging of the planets [3, 4, 7].

In [8] Sobczyk suggested an interpretation of special relativity in a complex vector-based language that is the natural generalization

of the Gibbs-Heaviside vector algebra of 3-dimensional space. Now here we want to extend Soboczyk's approach by considering mathematical formalisms (4) and (5) in the context of a complex 3-dimensional vector space $\mathbb{C}^3$. By following Soboczyk [8], we start by defining $\vec{b} = \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ as an orthonormal basis of this complex space equipped with a complex scalar product defined as

$$A \circ B = \sum_{k=1}^{3} \alpha_k \beta_k = \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3 \quad (7)$$

for $A = \sum_{k=1}^{3} \alpha_k \vec{e}_k \in \mathbb{C}^3$ and $B = \sum_{k=1}^{3} \beta_k \vec{e}_k \in \mathbb{C}^3$ where $\alpha_k, \beta_k \in C$.

In the approach based on the complex space the event horizon of an

inertial system is a subset
$\mathcal{H} \subset \mathbb{C}^3$:

$$\mathcal{H} = \left\{ \frac{X}{X} = \sum_{k=0}^{3} x_\mu \vec{e}_\mu \;\; where \;\; x_\mu \in \Re \right\} \quad (8)$$

and $x_\mu = x - v\tau$. Now, let $\mathcal{H}'$ be the event horizon of an inertial system moving along the x- axis with the velocity $v\vec{e}_1$ as seen by an observer in $\mathcal{H}$. Then the event horizons are related by the universal mapping

$$X' = Xexp(\phi e_1) \quad (9)$$

and the orthonormal rest frames by the boost $\vec{e}_k' = e^{-\frac{1}{2}\phi\vec{e}_1}\vec{e}_k e^{\frac{1}{2}\phi\vec{e}_1}$ for k=1,2,3 and the hyperbolic angle satisfies the following conditions

$$cosh\phi = \sqrt{1 - \frac{v^2}{c^2}} \quad (10)$$

$$senh\phi = \frac{v}{c}\sqrt{1 - \frac{v^2}{c^2}} \quad (11)$$

and thus
$\frac{v}{c} = tanh\phi.$
Suppose now that $\vec{X}(\tau) = \vec{X} - v\tau = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$ is the history of a particle moving in the inertial system $\mathcal{H}$ and that $\vec{X'}(\tau) = -v\tau + x'\vec{e}_1{}' + y'\vec{e}_2{}' + z'\vec{e}_3{}'$ is the history of the same particle in the inertial system $\mathcal{H}'$. The coordinates as measured in $\mathcal{H}$ are related to the corresponding coordinates as measured in $\mathcal{H}'$ by relations

$$e^{\frac{1}{2}\phi\vec{e}_1}X'e^{-\frac{1}{2}\phi\vec{e}_1} = e^{\frac{1}{2}\phi\vec{e}_1}Xe^{-\frac{1}{2}\phi\vec{e}_1} \quad (12).$$

This equation implies

$$-v\tau' + x'\vec{e}_1' = (-v\tau + x\vec{e}_1)(cosh\phi + \vec{e}_1 senh\phi) = cosh\phi(-v\tau + x\vec{e}_1)\left(1 + \vec{e}_1\frac{v}{c}\right) \quad (13)$$

and

$$y'\vec{e}_2' + z'\vec{e}_3' = y\vec{e}_2 + z\vec{e}_3 \quad (14)$$

which are the counterpart of the formalisms (8) and (9) in the complex space $\mathbb{C}^3$ and lead directly to Lorentz transformations. The physical consequence of these equations is therefore that the Minkowski space-time model $\mathbb{R}^4$ of special relativity is not fundamental but indeed is an emerging structure from a more fundamental complex space $\mathbb{C}^3$ where time exists merely as a mathematical quantity measuring the numerical order of material changes.

3. General Relativity in a Complex Space: mathematical formalism

Although General Relativity (GR) has got several experimental confirmations [9, 10], nevertheless it presents limits when strong gravitational fields are considered (which can lead to different interpretations of the sources of gravitational waves [11, 12]), as well as when one tries to search a compatibility with quantum theory.

In order to face some of the issues regarding general relativity, since the pioneering work of Jean-Emile Charon [13], several models have been developed by the theoretical physics community which are based on a complex space-time picture. For example, in order to treat the singularity at $r = 0$ of the Lorentzian Schwarzschild solution (which turns out to disappear on the real Riemannian section of the corresponding complexified space-time), in a series of papers [14-16] G. Esposito developed a version of complex general relativity where the Lorentzian space-time is replaced by a four-complex-dimensional complex-Riemannian manifold, with holomorphic connection and holomorphic curvature tensor. In this model, the Hamiltonian constraint is replaced by a geometric structure linear in the holomorphic multimomenta, and the mathematical formalism leads to some boundary conditions on two-complex-dimensional surfaces, which exhibit a link with the Penrose twistor programme.

More recently, in order to face the matter regarding the strong gravitational fields, Hess has proposed an algebraic extension of GR involving only real and pseudo-complex (called also hyper-complex) variables, which allows the investigation of the solutions for the limit of weak gravitational fields [17]. In Hess' pseudo-complex GR, the Einstein equations include an energy-momentum tensor, linked to vacuum fluctuations (dark energy), which assumes the form of an asymmetric ideal fluid [18]. In this approach, dark energy is chosen

in such a way that no event horizon appears and a general principle emerges, namely that mass not only curves the space (which leads to the standard GR) but also changes the vacuum properties in its vicinity, which in turn leads to an important deviation from the classical solution.

By following [19-22], in Hess' algebraic extension of GR one starts from a mapping of the real coordinates into the following pseudo-complex variables

$$X^{\mu} = x^{\mu} + Iy^{\mu} \quad (15)$$

where $I^2 = \pm 1$, x^{μ} are the standard coordinates in space-time, y^{μ} is the complex component. Equation (15) implies that, when $I^2 = -1$ one obtains complex variables, when $I^2 = 1$ one has pseudo-complex variables. Hess' theory implies that, if one makes the variation of the action

$$S = \int dx^4 \sqrt{-g}\,(\Re + 2\alpha) \quad (16),$$

where $\Re$ is the Riemann scalar and the last term corresponds to the cosmological constant, α being constant in order not to violate the Lorentz symmetry, one obtains the Einstein equations of motion for each component

$$G^{\pm}_{\mu\nu} - \frac{1}{2} g^{\pm}_{\mu\nu} R^{\pm}_{\mu\nu} = 8\pi T^{\pm}_{\mu\nu} \quad \text{(where here } c = G = 1) \quad (17).$$

In each component the mathematical structure of the theory has the form of a theory of general relativity, where all principles and symmetries are satisfied. Equations (17) include the effects of a minimal length parameter l – which is not measurable in present observations – in the sense that it leads to a pseudo-complex length element squared given by relation

$$d\omega^2 = g_{\mu\nu}[dx^{\mu}dx^{\nu} + l^2 du^{\mu}du^{\nu}] \quad (18)$$

which preserves Lorentz symmetry. Equations (17) turn out to have the same structure as the standard general relativistic equations, except for the fact that the energy-momentum tensor appearing on the right hand side describes an an-isotropic ideal fluid. As a consequence of the appearance of this non-zero energy-momentum tensor, the mass not only curves space-time but also changes vacuum properties. The vacuum fluctuations are associated with the energy-momentum tensor $T_{\mu\nu}$ and turn out to increase with the strength of the gravitational field. Hence, since problems arise when the strength of the gravitational field is large, Hess' model predicts that the behaviour near the horizon is not physical and requires a modification.

Now, here our aim is to provide a generalization of the pseudo-complex GR developed by Hess by using the model of the

three-dimensional (3D) dynamic quantum vacuum (DQV) developed by two of the authors of this paper (DF and AS) in a series of recent works [23-29]. We will call our generalization of the pseudo-complex general relativity as "complex relativity in a three-dimensional (3D) dynamic quantum vacuum (DQV)".

In our model, GR emerges as the hydrodynamic limit of some underlying theory of a more fundamental microscopic 3D quantum vacuum condensate where each elementary particle is determined by elementary reduction-state (**RS**) processes of creation/annihilation of virtual pairs particles corresponding to opportune changes of a fundamental quantum vacuum energy density. In this model, the curvature of space-time characteristic of GR emerges as a mathematical value of a more fundamental energy density of quantum vacuum. The changes and fluctuations of the quantum vacuum energy density generate a curvature of space-time similar to the curvature produced by a "dark energy" density, through a quantized metric characterizing the underlying microscopic geometry of the 3D quantum vacuum [23-29].

In our model, in epistemological affinity with the results obtained by Santos [30-32], the quantized metric of the 3D quantum vacuum condensate is expressed by relation

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu \quad (19)$$

where here the (quantum operators) coefficients of the metric are defined (in polar coordinates) as

$$\hat{g}_{00} = -1 + \hat{h}_{00},\ \hat{g}_{11} = 1 + \hat{h}_{11},\ \hat{g}_{22} = r^2\left(1 + \hat{h}_{22}\right),\ \hat{g}_{33} = r^2 \sin^2\vartheta\left(1 + \hat{h}_{33}\right),$$
$$\hat{g}_{\mu\nu} = \hat{h}_{\mu\nu} \text{ for } \mu \neq \nu \quad (20).$$

As regards the coefficients (20), multiplication of every term times the unit operator is implicit and, at the order $O\left(r^2\right)$, in the light of equation (6) one obtains

$$\left\langle \hat{h}_{\mu\nu} \right\rangle = 0 \text{ except } \left\langle \hat{h}_{00} \right\rangle = \frac{8\pi G}{3}\left(\frac{\Delta\rho_{qvE}}{c^2} + \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE} \right)^6 \right) r^2 \text{ and}$$

$$\left\langle \hat{h}_{11} \right\rangle = \frac{8\pi G}{3}\left(-\frac{\Delta\rho_{qvE}}{2c^2} + \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE} \right)^6 \right) r^2 \quad (21).$$

where the metric (21) is close to Minkowski (namely, when the distance $r \to \infty$, one has $\hat{g}_{\mu\nu} \to \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric) and $\Delta\rho_{qvE}^{DE}$ are opportune fluctuations of the quantum

vacuum energy density which determine the dark energy density on the basis of relation

$$\rho_{DE} \cong \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6 \quad (22)\ [23\text{-}29].$$

Here, therefore, dark energy emerges as energy itself of the 3D quantum vacuum. As a consequence of equation (22), in our approach of general relativity in a complex space, we propose that the fundamental arena is a pseudo-complex space $\mathbb{C}^4$ where the action (16) assumes the form

$$S = \int dx^4\sqrt{-g}\left(\Re + \frac{35Gc^2}{\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right) \quad (23).$$

Equation (23) represents the counterpart of the action (16) of Hess' model and its actual physical meaning at a more fundamental level. In other words, in our model, the action of the pseudo-complex space $\mathbb{C}^4$, which is the fundamental arena of general relativity, is directly associated with the fluctuations of the quantum vacuum energy density which mimic the action of dark energy.

By making the variations of the action (23) with respect to the metric $g_{\mu\nu}^{\pm}$, one obtains the following equations of motion

$$\Re_{\mu\nu}^{\pm} - \frac{1}{2}g_{\mu\nu}^{\pm}\Re_{\mu\nu}^{\pm} = 8\pi T_{\pm\,\mu\nu}^{\Lambda} \quad (24)$$

where

$$T_{\pm\,\mu\nu}^{\Lambda} = \lambda u_\mu u_\nu + \lambda\left(\dot{y}_\mu\dot{y}_\nu + u_\mu\dot{y}_\nu + u_\nu\dot{y}_\mu\right) + \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6 g_{\mu\nu}^{\pm} \quad (25).$$

In the light of equations (24)-(25), the fluctuations of the quantum vacuum energy density characterizing the pseudo-complex space, lead to the existence of a minimal length parameter given by relation

$$\Delta x \geq \frac{\hbar}{2\Delta p} + \frac{\Delta p}{2\hbar}\left(2\pi^2/3\right)^{2/3} l^{2/3} l_P^{4/3} \quad (26)$$

(where l_P is Planck length),
which can be considered as the uncertainty in the measure of the position.
Since the minimal length (26) is difficult to measure, at a first order of approximation, one can neglect this parameter and project the above equations into their real part. In this way, one obtains

$$\Re_{\mu\nu} - \frac{1}{2}g_{\mu\nu}^{\pm}\Re = 8\pi T_{\mu\nu,\Re}^{\Lambda} \quad (27)$$

where $T_{\mu\nu,\Re}^{\Lambda}$ is real and is given by

$$T_{\mu\nu,\Re}^{\Lambda} = \left(\rho_{\mathrm{DE}} + p_\vartheta^{\Lambda}\right)u_\mu u_\nu + p_\vartheta^{\Lambda} g_{\mu\nu} + \left(p_r^{\Lambda} - p_\vartheta^{\Lambda}\right)k_\mu k_\nu \quad (28)$$

where ρ_{DE} is dark energy density (given by (22)), p_ϑ^{Λ} and p_r^{Λ} are the tangential and pressure respectively, u_μ are the components of the

four-velocity of the virtual particles of the medium and k^μ is a space-like vector in the radial direction, which satisfies relation $u_\mu k^\mu = 0$. Because of the presence of y_μ the following relations hold

$$\lambda = 8\pi\left(p_\vartheta^\Lambda + \rho_{\mathrm{DE}}\right), \quad \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6 = 8\pi p_\vartheta^\Lambda, \quad \lambda y_\mu y_\nu = 8\pi\left(p_r^\Lambda - p_\vartheta^\Lambda\right)k_\mu k_\nu \quad (29).$$

Near the Schwarzschild radius the gravitational field is very strong and this condition corresponds to a singularity in the quantum vacuum energy density, which turns out to be proportional to the quantity $\frac{1}{\left(1-\frac{2V\Delta\rho_{qvE}}{c^2 r}\right)^2}$. As regards the fall-off of the negative energy density, which is finite at the Schwarzschild radius, in the light of the results obtained in [20], we can assume that it is a function of a parameter n and of the quantum vacuum energy density, i.e., proportional to $\frac{B_n}{r^n}$, where $B_n = b_n\left(\frac{V}{c^2}\Delta\rho_{qvE}\right)^n$ describes the coupling of the dark energy density (22) to the fluctuations of the quantum vacuum. In this way, the metric for the Kerr solution describing a rotating star is given by relations

$$g_{00} = -\frac{r^2 - 2\frac{V}{c^2}r\Delta\rho_{qvE} + a^2cos^2\vartheta + \frac{b_n\left(\frac{V}{c^2}\Delta\rho_{qvE}\right)^n}{(n-1)(n-2)r^{n-2}}}{r^2 + a^2cos^2\vartheta}$$

$$g_{11} = \frac{r^2 + a^2cos^2\vartheta}{r^2 - 2\frac{V}{c^2}r\Delta\rho_{qvE} + a^2cos^2\vartheta + \frac{b_n\left(\frac{V}{c^2}\Delta\rho_{qvE}\right)^n}{(n-1)(n-2)r^{n-2}}}$$

$$g_{22} = r^2 + a^2cos^2\vartheta$$

$$g_{33} = (r^2 + a^2)sen^2\vartheta + \frac{a^2sen^4\vartheta\left(2\frac{V}{c^2}r\Delta\rho_{qvE} - \frac{b_n\left(\frac{V}{c^2}\Delta\rho_{qvE}\right)^n}{(n-1)(n-2)r^{n-2}}\right)}{r^2 + a^2cos^2\vartheta}$$

$$g_{03} = \frac{-2asen^2\vartheta\frac{V}{c^2}r\Delta\rho_{qvE} + a\frac{b_n\left(\frac{V}{c^2}\Delta\rho_{qvE}\right)^n}{(n-1)(n-2)r^{n-2}}sen^2\vartheta}{r^2 + a^2cos^2\vartheta} \quad (30)$$

where $0 \leq a \leq \frac{V}{c^2}\Delta\rho_{qvE}$ is the counterpart, in our model, of the mass parameter of the Kerr solution and n=3,4,... The Schwarzschild solution is obtained for $a = 0$. Equations (30) become the standard Kerr solution of GR when $B_n = 0$.

For n=4 and $B_n = \frac{81}{8}$ one finds that an event horizon for $a = 0$ occurs at $\frac{3}{2}\frac{V}{c^2}\Delta\rho_{qvE}$. The event horizon is eliminated by imposing that the parameter B_n, in a given volume V of space, has a lower limit given by relation

$$B_n = \frac{2(n-1)(n-2)}{n}\left[\frac{2(n-1)}{n}\right]^{n-1}\left(\frac{V}{c^2}\Delta\rho_{qvE}\right)^n \quad (31).$$

As a consequence, an event horizon occurs at the coordinates

$$r_h = \frac{2(n-1)}{n}\frac{V}{c^2}\Delta\rho_{qvE} \quad (32)$$

which, for example, acquires the value $r_h = \frac{4}{3}\frac{V}{c^2}\Delta\rho_{qvE}$ for n=3, $r_h = \frac{3}{2}\frac{V}{c^2}\Delta\rho_{qvE}$ for n=4. In this situation, a mass function can be defined given by relation

$$m(r) = \left[1 - \frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right] \quad (33).$$

For the Schwarzschild regime the g_{00} component assumes the form $g_{00} = 1 - \frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}$, which reduces to the well-known expression in GR when the term proportional to $\frac{1}{r^3}$ in the mass (33) vanishes.

4. Predictions of the model of general relativity in complex space

Let us explore now the predictions of our model of "complex relativity in a 3D DQV", in particular as regards the motion of a material object in a circular orbit and the perihelion shift of Mercury.

As the regards the motion of a particle in a circular orbit, our model implies that the orbital frequency is given by

$$\omega_n = \frac{1}{a+\sqrt{\frac{2r}{h_n(r)}}} \quad (34)$$

where

$$h_n(r) = \frac{2}{r^2} - \frac{n\left(\frac{V}{c^2}\Delta\rho_{qvE}\right)^n}{(n-1)(n-2)r^{n+1}} \quad (35).$$

When B_n reaches its lower limit given by the right-hand side of equation (31), the quantity $h_n(r)$ assumes the form

$$h_3(r) = \frac{2}{r^4}\left(r^2 - \frac{16}{9}\right), \quad h_4(r) = \frac{2}{r^5}\left(r^3 - \frac{27}{8}\right) \quad (36).$$

In light of equations (36), one finds that, in our model of complex extension of general relativity, the orbital frequency of a material object in a circular orbit is always lower than the corresponding frequency predicted by GR, and that some differences only appear

near the Schwarzschild radius, and thus in situations which are near the size of a black hole, in agreement with the results obtained by Hess in [19-21]. Here, however, we can provide a deeper justification of this crucial result, namely in terms of the energy density fluctuations of the 3D DQV. In other words, in our approach of general relativity in a complex space, we can say that the orbital frequency of a material object in a circular orbit is always lower than the corresponding frequency predicted by GR and that some differences only appear near the Schwarzschild radius, as a consequence of more fundamental fluctuations of the energy density of the 3D DQV.

Moreover, in our approach, the orbital frequency fixes for B_n a maximum at

$$r_{\omega_{max}} = \left[\frac{n(n+2)b_{max}}{6(n-1)(n-2)}\right]^{\frac{1}{n-1}} \frac{V}{c^2}\Delta\rho_{qvE} \quad (37)$$

which is independent of the value of a. Equation (37) implies that for $n = 3$ the value is approximately $r = 1.72\frac{V}{c^2}\Delta\rho_{qvE}$ and for $n = 4$ it is $r = 1.89\frac{V}{c^2}\Delta\rho_{qvE}$ and therefore that the zero of the orbital frequency and the position of its maximum tend to grow with increasing n.

In analogy to the results obtained by Hess [20], one of the main predictions of our model is that the accretion disc emission will show both an outer dark ring and an inner bright ring. However, unlike Hess' model, here we can provide a deeper explanation of the formation of these outer dark ring and inner dark ring of the accretion disc emission, in terms of the fundamental fluctuations of the quantum vacuum energy density.

At the maximum of the orbital frequency – corresponding to opportune energy density fluctuations of the 3D DQV – neighbouring orbitals have nearly the same orbital frequency, circular orbits emerge and thus less friction and less heating are present. As a consequence of this, a dark ring appears in the accretion disc emission near the position $r_{\omega_{max}}$, which is 1.72 for $n = 3$. On the other hand, above and especially below the position of the maximum, the change in orbital frequency is large, the disc gets heated and, consequently, the gradient becomes significantly stronger and a bright ring takes place. The position of the maximum, which gives the position of the dark ring, is approximately the same for $n = 3$ and $n = 4$. For $b_n \to 0$, the curve is similar to the one for GR. Our model predicts a deviation for larger a with respect to the results of GR. At about $a = 0{,}45\frac{V}{c^2}\Delta\rho_{qvE}$ all orbits in our model are stable up to the

surface of the star, which turns out to lie at approximately $\frac{4}{3}\frac{V}{c^2}\Delta\rho_{qvE}$. For $a = \frac{V}{c^2}\Delta\rho_{qvE}$, in GR the last stable orbit is at $r = \frac{V}{c^2}\Delta\rho_{qvE}$. For lower values of $a = 0{,}45\frac{V}{c^2}\Delta\rho_{qvE}$ the particles reach further inside and due to the decrease of the potential, more energy is released, producing a brighter disc.

In our model, the reason for the dark ring and bright ring of the accretion disc emission lies in the variability of the dragging effect caused by the energy density fluctuations of the 3D DQV. A possibility to experimentally distinguish standard general relativity and our model of general relativity in a complex space is thus represented by the observation of the differences in the structure of an accretion disc determined by the quantum vacuum energy density fluctuations.

On the other hand, a further prediction of our model is that the surface of the horizon of events is given by

$$r_h = \frac{2(n-1)}{n}\frac{V}{c^2}\Delta\rho_{qvE} \quad (38).$$

This parameter can be put in correlation with gravitational wave analysis (see, for example, [33]) and with observations of the Event Horizon Telescope [34]. Our model predicts thus that, in the Schwarzschild case (a = 0), the size of the black hole is smaller than in the standard theory while, in the Kerr case with $a = 1\frac{V}{c^2}\Delta\rho_{qvE}$, it turns out to be larger.

Let us analyse now the predictions of our model of general relativity in a complex space as regards the perihelion shift of Mercury, the first and most famous experimental test of general relativity. In this regard, our aim is to show that GR gives the dominant contribution, while the additional ones from its complex extension are too low to be measured (a result that is compatible with the fact that the gravitational field in the solar system is simply too weak, and in agreement with the results obtained by Hess in [19-21]).

In our model, one starts from the following real length element in the Schwarzschild solution, depending of the quantum vacuum energy density fluctuations

$$ds^2 = -\left(1 - \frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)dt^2 + \left(1 - \frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2) \quad (39).$$

If one divides (39) by ds^2 and uses the definition $L = -1$ for the Lagrangian suggested in [28], one obtains

$$1=\left(1-\frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)\dot{t}^2-\left(1-\frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)^{-1}\dot{r}^2-$$
$$r^2\left(\dot{\theta}^2+sin^2\theta\dot{\varphi}^2\right) \quad (40)$$

where the dot refers to the derivative in s. By imposing the angular momentum conservation $r^2\dot{\varphi}=const$, that the motion takes place in the plane $\theta=\frac{\pi}{2}$ and that $\left(1-\frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)\dot{t}=h=const$, equation (40) yields

$$1=-\left(1-\frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)^{-1}h^2-\left(1-\frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)^{-1}\dot{r}^2-\frac{\Delta x^2}{r^2}$$
(41),

where Δx is given by (28).

Now, if one changes the derivative with respect to s to the one with respect to φ (which we denote with r') and then multiplying (41) by $\left(1-\frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)$, after some mathematical manipulations, the following relation is obtained

$$\left(1-\frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)=h^2-\frac{\Delta x^2}{r^4}(r')^2-\frac{\Delta x^2}{r^2}\left(1-\frac{2\left[1-\frac{27}{32}\frac{V^3}{r^3c^6}\Delta\rho_{qvE}^3\right]}{r}\right)$$
(42).

Then, by defining the variable $u=\frac{1}{r}$, after some mathematical manipulations, one arrives at equation

$$u''+u=\frac{V\Delta\rho_{qvE}}{c^2\Delta x^2}+3\frac{V\Delta\rho_{qvE}}{c^2}u^2-\frac{27}{8}\frac{V^4\Delta\rho_{qvE}^4}{c^8\Delta x^2}u^3-\frac{81}{16}\frac{V^4\Delta\rho_{qvE}^4}{c^8\Delta x^2}u^5 \quad (43).$$

Now, in the 3D quantum vacuum of the complex space, the **RS** processes corresponding to elementary fluctuations of the quantum vacuum energy density determine a motion of virtual particles endowed with frequencies

$$\omega=\frac{2\Delta\rho_{qvE}V}{\hbar n} \quad (44)$$

and thus give rise to a total velocity of the quantum vacuum

$$v_{qv}=\frac{2\Delta\rho_{qvE}V}{\hbar n}R \quad (45)$$

namely

$$v_{qv}=\frac{2Ac^2\Delta x^2}{\hbar n}R \quad (46)$$

in each given volume into consideration, where R is the distance from the centre of that volume and the quantity

$$A = \frac{V\Delta\rho_{qvE}}{c^2\Delta x^2} \quad (47)$$

can be defined as the areal velocity on the planetary plane. As a consequence, in light of the dependence between velocity of the quantum vacuum and areal velocity on the planetary plane

$$A = \frac{\hbar n v_{qv}}{2c^2\Delta x^2 R} \quad (48)$$

equation (43) may be conveniently expressed as

$$u'' + u = A + \frac{2c^2\Delta x^2 R\varepsilon}{\hbar n v_{qv}} u^2 - \frac{A}{8}\left(\frac{2c^2\Delta x^2 R\varepsilon}{\hbar n v_{qv}}\right)^3 u^3 - \frac{1}{16}\left(\frac{2c^2\Delta x^2 R\varepsilon}{\hbar n v_{qv}}\right)^4 u^5 \quad (49)$$

where the number $\varepsilon = 3\frac{V^2\Delta\rho_{qvE}^2}{c^4}\frac{1}{\Delta x^2}$ is very small in case of the solar system, namely of the order 10^{-7}. In equation (49), the first term on the right hand side corresponds to the classical, Newtonian contribution, while the second term reproduces the standard relativistic correction which leads to the known perihelion shift of Mercury in function of the velocity of the quantum vacuum. The next terms are explicit new contributions – always associated with the velocity of the 3D DQV – which arise from the complex space of GR and, since ε is of the order of 10^{-7} in the solar system, these deviations can be hardly detected in solar system observations, where therefore the standard view of GR and our interpretation of GR in complex space can be considered indistinguishable from each other.

5. Conclusions and perspectives

In this paper a new interpretation of special relativity and general relativity in a complex space has been proposed. The model developed in this paper suggests that the fundamental arena of special relativity is a complex $\mathbb{C}^4$ space which is time-invariant in the sense that time is not its dimension but exists merely as a mathematical quantity measuring the numerical order of material changes (and which can be defined as the proper time of the observer under consideration) and the physical duration of motion can be defined as the "true" physical scale for the numerical order. Here, the Minkowski space-time model is indeed an emerging structure from the more fundamental complex space $\mathbb{C}^3$.

As regards general relativity, in our model the fundamental arena is a pseudo-complex space $\mathbb{C}^4$ where the action is directly associated with the elementary energy density fluctuations of the 3D quantum vacuum which occur in correspondence to RS processes of

creation/annihilation of virtual particles. The model of "complex relativity in a 3D space" here introduced predicts that, as a consequence of the behaviour of the fluctuations of the quantum vacuum energy density, the orbital frequency of a material object in a circular orbit is always lower than the corresponding frequency predicted by general relativity, and that some differences only appear near the Schwarzschild radius. Moreover, as regards the perihelion shift of Mercury, it is completely coherent with the results of general relativity and provides a justification of these results in terms of a fundamental velocity of the quantum vacuum.

On the other hand, Gödel's development of general relativity shows that Einstein field equations lead to the contradiction, namely, one could move back in time and kill his grandfather and so he could not be born. He understood that his development of General Relativity proves that time has no physical existence and nobody can travel in time. Still today he is misunderstood by thinking that his work is proving that time travel is possible [35]. The idea that the fundamental arena of general relativity is a complex space throws new perspectives of solution to this paradox of time travel: nobody can travel in time because universal space is time-invariant.

Finally, it must be emphasized that there are several issues that have to be addressed in this model, in particular as regards cosmology. In this regards, the perspective is opened that the Friedmann equation, which combine the GR field equations and the Friedmann-Lemaitre-Robertson-Walker (FRLW) metric describing a homogeneous, isotropic universe, with matter and energy uniformly distributed as a perfect fluid, can be expressed in the form

$$H^2(a) = \frac{8\pi G}{3}\left(V\Delta\rho_{qvE}a^{-3} + \rho_{rad}a^{-4} + \frac{35Gc^2}{\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6 e^{-3\int_a^1[1+\omega(a')]dlna'}\right) - \frac{kc}{a^2} \quad (50)$$

where $\Delta\rho_{qvE}$ are the fluctuations of the quantum vacuum energy density corresponding to the appearance of matter, ρ_{rad} is the density of the quantum vacuum associated with radiation, which satisfies relation

$$E_{rad} = \rho_{rad}c^2 \quad (51),$$

a is the dimensionless scale factor describing the evolution of the universe, $\omega = \frac{p}{\Delta\rho_{qvE}c^2}$, p is the pressure, H is the Hubble parameter, and k is a parameter which for $k = 0$ corresponds to a flat universe. Equation (50) implies that, as a consequence of the behaviour of the quantum vacuum energy density, the early universe was radiation-

dominated, until the temperature dropped enough for matter density to being to dominate. The energy density of dark energy is constant if its equation of state parameter $\omega = 1$. Because the matter energy density drops as the scale factor increased, the fluctuations of the quantum vacuum $\Delta\rho_{qvE}^{DE}$ associated with the action of dark energy began to dominate in the recent past and, at the present time $(a(t) = 1)$, we live in a universe dominated by these type of fluctuations of the vacuum [36]. As regards the consequences of the modified Friedmann equation (50) regarding the behaviour of the scale factor of the universe, further research will give you more information.

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